D. 3

a) Let X be the average wind speed in an hour.

X has distribution function

$$F(x) = 1 - e^{-x^2/100}, \quad x \geq 0.$$

$\hookrightarrow$ $$P(3 \leq X \leq 25) = F(25) - F(3)$$
$$\approx 0{,}922.$$

b) Let Y be the number of hours in a day that power is produced.

Assuming independence, we have $Y \sim Bin(24;\ 0{,}922)$. Then

$$P(Y \geq 20) = P(Y=20) + P(Y=21) + \dots + P(Y=24)$$

$$= \binom{24}{20} 0.922^{20} (1-0.922)^{24-20} + \dots$$

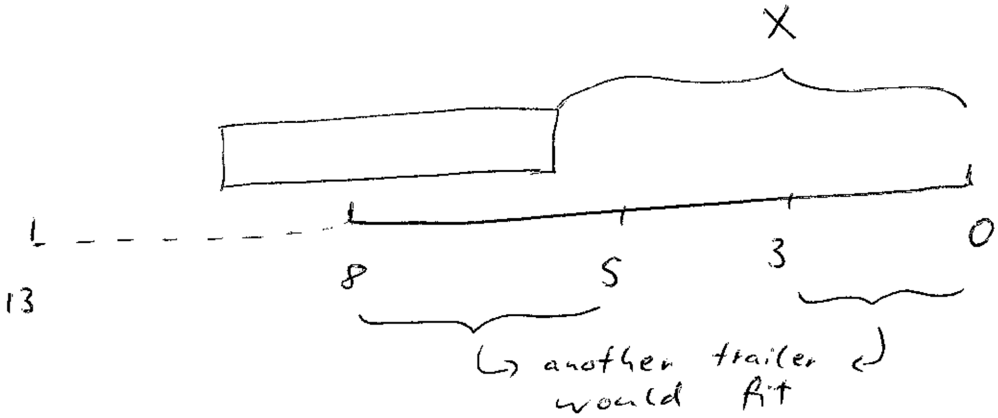
$$+ \binom{24}{24} 0.922^{24} (1-0.922)^{24-24}$$

$$\approx 0{,}965$$

The assumption of independence is very doubtable here. However, we can not compute anything without knowledge about structure of dependence.

D. 4

Let X be the free space behind the trailer.



a) X can take any values between 0 and 8 (meter). Hence, X is continuous.

Another trailer would fit for $\underbrace{0 \le X \le 3}_{\text{before 1st trailer}}$ and $\underbrace{5 \le \cancel{X} \le 8}_{\text{behind 1st trailer}}$.

b) X is assumed to be uniform distributed on $[0, 8]$, i.e., X has density $f(x) = \frac{1}{8}$, $0 \le x \le 8$.

Hence,

$$P(0 \le X \le 3 \text{ or } 5 \le X \le 8)$$

$$= P(0 \le X \le 3) + P(5 \le X \le 8)$$

$$= \int_0^3 1/8 \, dx + \int_5^8 1/8 \, dx$$

$$= 3/8 + 3/8 = 3/4$$ //

0.5

Let X be the weight of a randomly choosen person. Assume $X \sim \mathcal{N}(\mu, \sigma^2)$.

There is no right or wrong about the choice of μ and σ^2, only more and less reasonable!

Based on the rule of thumb

$$P(\mu - 2\sigma \leq X \leq \mu + 2\sigma) \approx 95\%$$

we assume $\mu = 75$ kg and $\sigma = 12{,}5$ kg.

Then ca. 95% of all persons have a weight between 50 and 100 kg which seems reasonable.

Now let X_i be the weight of the i-th person. We assume that the persons have independent weights. Then

$$Y = X_1 + X_2 + \ldots + X_8 \sim \mathcal{N}(8 \cdot 75;\ 8 \cdot 12{,}5^2)$$

$$= \mathcal{N}(600, 1250).$$

We get $\mathbb{P}(Y > 630) = 1 - P\Big(\underbrace{\frac{Y-600}{\sqrt{1250}}}_{\sim \mathcal{N}(0,1)} \leq \underbrace{\frac{630-600}{\sqrt{1250}}}_{\approx 0{,}85} \Big)$

$$\approx 1 - \Phi(0{,}85) = 0{,}20$$

D. 6

Let Y be the number of successful experiments. We have $Y \sim Bin(100; \frac{2}{10})$.

a) $P(Y \leq 18)$ is hard to compute by hand.

R gives us the solution via

pbinom (18, 100, 2/10) $\Rightarrow$ 0,362 //

b) Let $X_1; \dots; X_{100} \sim Bin(1, 2/10)$, independent.

Then $Y \overset{d}{=} X_1 + \dots + X_{100}$.

$\overset{d}{=}$ equal in distribution.

We have $\mu = EX_i = 1 \cdot 2/10 = 0.2$ and

$\sigma^2 = Var\, X_i = 1 \cdot 2/10 \cdot (1 - 2/10) = 0.16$.

Central limit theorem / CLT / ZGS:

$$\frac{\sum_{i=1}^{n} X_i - n\mu}{\sigma \sqrt{n}} \approx \mathcal{N}(0,1) \quad \text{for large } n.$$

Hence,

$$P(Y \leq 18) = P\left(\sum_{i=1}^{100} X_i \leq 18\right) =$$

$$= P\left(\underbrace{\frac{\sum_{i=1}^{100} X_i - 100 \cdot 0.2}{\sqrt{100} \cdot \sqrt{0.16}}}_{\approx \mathcal{N}(0,1)} \leq \underbrace{\frac{18 - 100 \times 0{,}2}{\sqrt{100} \cdot \sqrt{0.16}}}_{= -0{,}5} \right)$$

$$\approx \Phi(-0.5) = 1 - \Phi(0.5)$$

$$= 0{,}309 \quad //$$

c) With continuity correction

$$P(Y \leq 18) = P(Y \leq 18{,}5)$$

$$= \ldots \text{ in the same way} \ldots$$

$$= P\left(\underbrace{\frac{\sum_{i=1}^{100} X_i - 100 \cdot 0.2}{\sqrt{100} \cdot \sqrt{0.16}}}_{\approx \mathcal{N}(0,1)} \leq \underbrace{\frac{18{,}5 - 100 \cdot 0.2}{\sqrt{100} \cdot \sqrt{0.16}}}_{= -0{,}375} \right)$$

$$\approx \Phi(-0{,}375) = 1 - \Phi(0{,}375)$$

$$= 0{,}354 \quad //$$